Microeconomic Theory I: Study Note

Quarter 1: Preference, Choice, and Utility

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2 Stochastic Choices

Roadmap: Random choice rule → Random Utility Representation → Discrete Choice special case → Regularity and Block–Marschak → Luce's IIA and Logit → Identification.

Classical choice theory assumes the analyst observes deterministic decisions. Stochastic choice theory relaxes this: the *decision-maker* knows their payoff, but the *analyst* observes only frequencies. The framework asks when those observed frequencies are consistent with some underlying random utility.

2.1 Random Choice Rules and Random Utility

Definition: Random Choice Rule

$P(x, A)$ is a *random choice rule*: the probability that item $x \in A$ is chosen from menu A, equivalently the observed frequency with which x is chosen from A.

For it to be a proper probability:

- $\sum_{x \in A} P(x, A) = 1$ Total probability
- $\sum_{x \notin A} P(x, A) = 0$ Measure 0

Definition: Random Utility and Representation

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. A *random utility function* is a map $u : \Omega \to \mathbb{R}^X$, where for each state ω, $u(\omega, \cdot) : X \to \mathbb{R}$ is the agent's utility in that state.

The *winning event* is $S(x, A) := \{\omega : x \text{ is preferred to all } y \in A \text{ at } \omega\}$: *Those ω in which x is preferred amongst all in A*

P has a **random utility representation** if $\exists\, (\Omega, \mathcal{F}, \mathbb{P})$ and $u : \Omega \to \mathbb{R}^X$ such that $P(x, A) = \mathbb{P}[S(x, A)]$ for all $x \in A$, $A \subseteq X$.

Example: $X = \{a, b\}$. State ω: $u(\omega, a) = 5$, $u(\omega, b) = 2$. State ω': $u(\omega', a) = 1$, $u(\omega', b) = 4$. The agent's preference flips across states.

2.2 Discrete Choice as a Special Case

Definition: Discrete Choice Model

Assume X is discrete. There exists a deterministic $V : X \to \mathbb{R}$ (average preference) and a random utility shock $\widetilde{\varepsilon} : \Omega \to \mathbb{R}^X$ with continuous density and full support. Final utility in state ω:

$$u(\omega, x) = V(x) + \widetilde{\varepsilon}(\omega, x), \qquad \mathbb{E}[\widetilde{\varepsilon}] = 0.$$

The winning event becomes $S(x, A) = \{\omega : V(x) + \widetilde{\varepsilon}(\omega, x) \geq V(y) + \widetilde{\varepsilon}(\omega, y),\ \forall y \in A\}$.

Definition: Positivity

P satisfies **positivity** if $P(x, A) > 0$ for all $x \in A$ (all positive support)

*0 support means never considered. Mathematically it is troublesome as Logit cannot be used directly. Need to partition into only positive support items

Proposition: R.U. ⇔ Discrete Choice (Prop. 1.3.1)

Random utility $\iff$ Discrete choice when $|X| < \infty$ and $P > 0$.
Special cases:

- $\tilde{\varepsilon} \sim \text{Gumbel}(0) \Rightarrow$ **Logit** choice rule.
- $\tilde{\varepsilon} \sim \mathcal{N}(0, 1) \Rightarrow$ **Probit** choice rule.

2.3 Regularity and Block–Marschak

Definition: Regularity

P satisfies *regularity* if for all $x \in A \subseteq B$:

$$P(x, A) \ \geq \ P(x, B).$$

Enlarging the menu weakly decreases the probability of choosing x.

Theorem: R.U. Implies Regularity (Thm. 1.3.1)

If P has a random utility representation, then P satisfies regularity. The converse does *not* hold in general.

Proof

$P(x, A) = \mathbb{P}[S(x, A)]$. For $x \in A \subseteq B$: $S(x, B) \subseteq S(x, A)$ (winning against all of B implies winning against all of A). By monotonicity of probability: $P(x, B) \leq P(x, A)$. ■

Theorem: Block–Marschak (Thm. 1.3.2)

If $|X| \leq 3$ and P satisfies regularity, then there exists a random utility representation for P.
More generally, a random utility representation exists if and only if the Block–Marschak inequalities hold: for all x and $A \subseteq X$,

$$\sum_{B \supseteq A} (-1)^{|B \setminus A|} P(x, B) \ \geq \ 0.$$

Conditional converse on Theorem 1.3.1

2.4 Luce's Representation and IIA

Definition: Luce's Representation and IIA

P has a **Luce representation** if there exists $w : X \to \mathbb{R}_{++}$ such that

$$P(x, A) = \frac{w(x)}{\sum_{y \in A} w(y)}.$$

A functionally weighted probability mass. $w(x)$: attractiveness weight of x. So Luce form is relative attractiveness of x

ex. logit form: $w(x) = e^{u(x)}, P(i \mid \{i, j\}) = \frac{e^{V(i)}}{e^{V(i)} + e^{V(j)}}$.

P satisfies **Luce's IIA** if for all $x, y \in A \cap B$ chosen with positive probability:

$$\frac{P(x, A)}{P(y, A)} = \frac{P(x, B)}{P(y, B)}.$$

Menus do not matter for *relative* choice probabilities. Luce's IIA is strictly stronger than regularity.

Theorem: Luce's Theorem (Thm. 1.3.1′)

P has a Luce representation if and only if P satisfies

- Luce's IIA
- Positivity - Not blowing up the denominator of Luce rep

Proof sketch

($\Rightarrow$) If P has Luce representation w: $\frac{P(x,A)}{P(y,A)} = \frac{w(x)}{w(y)}$, independent of A. Positivity follows from $w > 0$.

($\Leftarrow$) Fix $x_0 \in X$, set $w(x_0) = 1$. For any x, IIA on $\{x, x_0\}$ gives $w(x) = \frac{P(x,\{x,x_0\})}{P(x_0,\{x,x_0\})} > 0$. Verify $P(x, A) = w(x)/\sum_{y \in A} w(y)$ for all menus by inductive application of IIA. ■

Red Bus–Blue Bus Paradox

$X = \{t, bb, rb\}$ (train, blue bus, red bus). $P(t, \{t, bb\}) = P(t, \{t, rb\}) = \frac{1}{2}$ and $P(bb, \{bb, rb\}) = \frac{1}{2}$.

By Luce's representation: $w(t) = w(bb) = w(rb)$, so $P(t, X) = P(bb, X) = P(rb, X) = \frac{1}{3}$.

But intuitively the two buses are near-identical substitutes, so the natural split is $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$. IIA cannot accommodate substitutability patterns — this is the model's fundamental limitation.

2.5 Identification in Stochastic Choice

Proposition: Identification Failure and Recovery

In general: identification fails in stochastic choice. The utility distribution cannot be recovered from observed choice frequencies alone.

Under i.i.d. discrete choice (Assumption I): $u(\omega, x) = V(x) + \varepsilon_x$, $\varepsilon_x \sim F$ i.i.d. If (V_1, ε) and (V_2, ε) are two representations for the same P, then $V_1(x) = V_2(x) + k$ for all $x \in X$ (some constant $k \in \mathbb{R}$). The average utility V is identified *up to an additive constant.*

2.6 Summary

You observe choice probabilities instead of hard, 100% choices.

- **Random Utility Model**: agent gets *random* shocks $u(\cdot)$, then ex-post makes *deterministic* choice $u(\omega, \cdot)$
- **RUM** $\Rightarrow$ **regularity** ($A \subseteq B \Rightarrow P(x, A) \geq P(x, B)$): Larger menu shrinks the probability of x getting chosen
- **Luce** is a special type of RUM where shocks are i.i.d. (Logit: i.i.d. Gumbel; Probit: i.i.d. Normal). Luce gives $v(x)$ as **attractiveness** of x relative to other choices.
- **Luce IIA**: Probability ratio (relative attractiveness) independent of menu changes.