Microeconomic Theory I: Study Note

Quarter 1: Preference, Choice, and Utility

Instructor: Jonathan Libgober

Tianqi Zhang

Department of Economics

University of Southern California

Fall 2025

6 Choice under Uncertainty

Roadmap: Setting (simple and compound lotteries) → Independence axiom → EU representation (vNM) → Uniqueness up to affine transformation → Violations of EU.

6.1 The Setting: Lotteries

Definition: Simple and Compound Lotteries

Let C be a finite set of outcomes with $|C| = n$. A *simple lottery* is $L = (p_1, \ldots, p_n)$ with $p_i \geq 0$ and $\sum_i p_i = 1$. The set of all simple lotteries is the $(n-1)$-simplex $\Delta(C)$.
A *compound lottery* $\{(L_j, \alpha_j)\}_{j=1}^{J}$ with $\sum_j \alpha_j = 1$ reduces to the simple lottery with $\Pr[\text{outcome } i] = \sum_j \alpha_j p_i^j$. Decision-makers are assumed to reduce compound lotteries.

6.2 Independence Axiom and Its Consequences

Definition: Independence Axiom

$\succsim$ over $\Delta(C)$ satisfies *independence* if for all $L, L', L'' \in \Delta(C)$ and $\alpha \in (0,1)$:

$$L \succsim L' \iff \alpha L + (1-\alpha)L'' \succsim \alpha L' + (1-\alpha)L''.$$

Mixing with any third lottery L'' does not change the ordering. *This is the key behavioral restriction*: violations (Allais, Ellsberg) are the source of most well-known paradoxes.

Proposition: Indifference Curves are Parallel under Independence

For $L \sim L'$, independence implies all convex combinations $\alpha L + (1-\alpha)L'$ are indifferent to L. Consequently, indifference curves in the simplex are *straight* and *parallel* $\iff$ Independence.

Proof

$L \sim L'$ and independence with $L'' = L'$: $\alpha L + (1-\alpha)L' \sim \alpha L' + (1-\alpha)L' = L'$. Transitivity: $\alpha L + (1-\alpha)L' \sim L \sim L'$. For parallelism: two distinct indifference curves are both affine (straight) by the above. If non-parallel, they intersect at some L^*, collapsing two distinct indifference classes — contradiction. ■

6.3 Expected Utility Representation

Definition: Expected Utility (vNM)

A preference $\succsim$ over $\Delta(C)$ has an *expected utility representation* if $\exists\, u : C \to \mathbb{R}$ such that for any $L = (p_1, \ldots, p_n)$:

$$U(L) := \sum_i p_i\, u(c_i).$$

U is *linear in probabilities.* u is the **Bernoulli utility function** (utility over outcomes); U is the **vNM utility function** (utility over lotteries).
Uniqueness up to affine transformation: if u and $\tilde{u}$ are two EU representations of the same $\succsim$, then $u = \beta + \gamma\tilde{u}$ for some $\beta \in \mathbb{R}$, $\gamma > 0$. This is **cardinal** uniqueness: the curvature of u carries meaning (risk attitudes), unlike ordinal utility.

Theorem: von Neumann–Morgenstern Theorem

$\succsim$ over $\Delta(C)$ is rational, continuous, and satisfies independence if and only if it admits an expected utility representation.

Proof sketch ($\Leftarrow$)

Given rationality and continuity, there exist best and worst lotteries $\overline{L}$ and $\underline{L}$. By continuity, $\forall\, L\ \exists!\, \lambda_L \in [0,1]$ s.t. $L \sim \lambda_L \overline{L} + (1-\lambda_L)\underline{L}$. Set $U(L) = \lambda_L$. Independence ensures U is linear in probabilities. Restricting to degenerate lotteries δ_{c_i} gives Bernoulli values $u(c_i) = U(\delta_{c_i})$. ■

6.4 Well-Known Violations of Expected Utility

Ellsberg's Paradox — Ambiguity Aversion

Two urns: Urn 1 has 50 black, 50 white. Urn 2 has unknown proportions. Most subjects prefer Urn 1 for both \$100-if-white and \$100-if-black bets. Under EU: Choice 1 gives $p < \frac{1}{2}$; Choice 2 gives $p > \frac{1}{2}$. No consistent p exists. Source: *ambiguity aversion* — aversion to unknown probabilities, not just risk.

Allais Paradox — Common Ratio Effect

Bet 1: most prefer \$2400 certain over $(\$2500, 0.33;\ \$2400, 0.66;\ \$0, 0.01)$. Under EU: $0.34\,U(2400) > 0.33\,U(2500)$.
Bet 2: most prefer $(\$2500, 0.33;\ \$0, 0.67)$ over $(\$2400, 0.34;\ \$0, 0.66)$. Under EU: $0.33\,U(2500) > 0.34\,U(2400)$.
These contradict each other. People overweight the certainty of \$2400 in Bet 1 but not in Bet 2.

Machina's Paradox — Unrealized Outcomes

A: Venice with $\Pr = 0.99$; film about Venice with $\Pr = 0.01$. B: Venice with $\Pr = 0.99$; nothing with $\Pr = 0.01$. Most prefer A: wellbeing depends on *unrealized* outcomes, violating reduction of compound lotteries underlying EU.

Jonathan's Paradox — Preference for Fairness

1 toy, 2 children. A parent may strictly prefer a 50–50 lottery over giving to either child, due to *procedural fairness*. But under EU, if $U(\text{child 1 gets toy}) = U(\text{child 2 gets toy})$ (symmetry), then $U(\text{lottery}) = \frac{1}{2}U(1) + \frac{1}{2}U(2) = U(1)$, so the lottery cannot be strictly preferred. EU cannot

accommodate strict preference for randomization as a fairness device.